Standards for Reporting the Optical Aberrations of Eyes

Larry N. Thibos,¹ Raymond A. Applegate,² James T. Schwiegerling,³ Robert Webb⁴, and VSIA Standards Taskforce Members

¹School of Optometry, Indiana University, Bloomington, IN 47405, ²Department of Ophthalmology, University of Texas Health Science Center at San Antonio, San Antonio, TX 78284, ³Department of Ophthalmology, University of Arizona, Tucson, AZ 85721, ⁴Schepens Research Institute, Boston, MA 02114 thibos@indiana.edu, Applegate@uthscsa.edu, jschwieg@u.arizona.edu, webb@helix.mgh.harvard.edu

Abstract: In response to a perceived need in the vision community, an OSA taskforce was formed at the 1999 topical meeting on vision science and its applications (VSIA-99) and charged with developing consensus recommendations on definitions, conventions, and standards for reporting of optical aberrations of human eyes. Progress reports were presented at the 1999 OSA annual meeting and at VSIA-2000 by the chairs of three taskforce subcommittees on (1) reference axes, (2) describing functions, and (3) model eyes. The following summary of the committee's recommendations is available also in portable document format (PDF) on OSA Optics Net at http://www.osa.org/. **OCIS codes:** (330.0330) Vision and color; (330.5370) Physiological optics

Background

The recent resurgence of activity in visual optics research and related clinical disciplines (e.g. refractive surgery, ophthalmic lens design, ametropia diagnosis) demands that the vision community establish common metrics, terminology, and other reporting standards for the specification of optical imperfections of eyes. Currently there exists a plethora of methods for analyzing and representing the aberration structure of the eye but no agreement exists within the vision community on a common, universal method for reporting results. In theory, the various methods currently in use by different groups of investigators all describe the same underlying phenomena and therefore it should be possible to reliably convert results from one representational scheme to another. However, the practical implementation of these conversion methods is computationally challenging, is subject to error, and reliable computer software is not widely available. All of these problems suggest the need for operational standards for reporting aberration data and to specify test procedures for evaluating the accuracy of data collection and data analysis methods.

Following a call for participation [1], approximately 20 people met at VSIA-99 to discuss the proposal to form a taskforce that would recommend standards for reporting optical aberrations of eyes. The group agreed to form three working parties that would take responsibility for developing consensus recommendations on definitions, conventions and standards for the following three topics: (1) reference axes, (2) describing functions, and (3) model eyes. It was decided that the strategy for Phase I of this project would be to concentrate on articulating definitions, conventions, and standards for those issues which are not empirical in nature. For example, several schemes for enumerating the Zernike polynomials have been proposed in the literature. Selecting one to be the standard is a matter of choice, not empirical investigation, and therefore was included in the charge to the taskforce. On the other hand, issues such as the maximum number of Zernike orders needed to describe ocular aberrations adequately is an empirical question which was avoided for the present, although the taskforce may choose to formulate recommendations on such issues at a later time. Phase I concluded at the VSIA-2000 meeting.

Reference Axis Selection

Summary

It is the committee's recommendation that the ophthalmic community use the line-of-sight as the reference axis for the purposes of calculating and measuring the optical aberrations of the eye. The rationale is that the line-of-sight in the normal eye is the path of the chief ray from the fixation point to the retinal fovea. Therefore, aberrations measured with respect to this axis will have the pupil center as the origin of a Cartesian reference frame. Secondary lines-of-sight may be similarly constructed for object points in the peripheral visual field. Because the exit pupil is not readily accessible in the living eye whereas the entrance pupil is, the committee recommends that calculations for specifying the optical aberration of the eye be referenced to the plane of the entrance pupil.

Background

Optical aberration measurements of the eye from various laboratories or within the same laboratory are not comparable unless they are calculated with respect to the same reference axis and expressed in the same manner. This requirement is complicated by the fact that, unlike a camera, the eye is a decentered optical system with non-rotationally symmetric components (Fig. 1). The principle elements of the eye's optical system are the cornea, pupil, and the crystalline lens. Each can be decentered and tilted with respect to other components, thus rendering an optical system that is typically dominated by coma at the foveola.



Fig. 1. The cornea, pupil, and crystalline lens are decentered and tilted with respect to each other, rendering the eye a decentered optical system that is different between individuals and eyes within the same individual.

The optics discipline has a long tradition of specifying the aberration of optical systems with respect to the center of the exit pupil. In a centered optical system (e.g., a camera, or telescope) using the center of the exit pupil as a reference for measurement of on-axis aberration is the same as measuring the optical aberrations with respect to the chief ray from an axial object point. However, because the exit pupil is not readily accessible in the living eye, it is more practical to reference aberrations to the entrance pupil. This is the natural choice for objective aberrometers which analyze light reflected from the eye.

Like a camera, the eye is an imaging device designed to form an in-focus inverted image on a screen. In the case of the eye, the imaging screen is the retina. However, unlike film, the "grain" of the retina is not uniform over its extent. Instead, the grain is finest at the foveola and falls off quickly as the distance from the foveola increases. Consequently, when viewing fine detail, we rotate our eye such that the object of regard falls on the foveola (Fig. 2). Thus, aberrations at the foveola have the greatest impact on an individual's ability to see fine details.



Fig. 2. An anatomical view of the macular region as viewed from the front and in cross section (below). a: foveola, b: fovea, c: parafoveal area, d: perifoveal area. From *Histology of the Human Eye* by Hogan. Alvarado Weddell, W.B. Sauders Company publishers, 1971, page 491.

Two traditional axes of the eye are centered on the foveola, the visual axis and the line-of-sight, but only the latter passes through the pupil center. In object space, the visual axis is typically defined as the line connecting the fixation object point to the eye's first nodal point. In image space, the visual axis is the parallel line connecting the second nodal point to the center of the foveola (Fig. 3, left). In contrast, the line-of-sight is defined as the (broken) line passing through the center of the eye's entrance and exit pupils connecting the object of regard to the foveola (Fig. 3, right). The line-of-sight is equivalent to the path of the foveal chief ray and therefore is the axis which conforms to optical standards. The visual axis and the line of sight are not the same and in some eyes the difference can have a large impact on retinal image quality [2]. For a review of the axes of the eye see [3]. (To avoid confusion, we note that Bennett and Rabbetts [4] re-define the visual axis to match the traditional definition of the line of sight. The Bennett and Rabbetts definition is counter to the majority of the literature and is not used here.)

When measuring the optical properties of the eye for objects which fall on the peripheral retina outside the central fovea, a secondary line-of-sight may be constructed as the broken line from object point to center of the entrance pupil and from the center of the exit pupil to the retinal location of the image. This axis represents the path of the chief ray from the object of interest and therefore is the appropriate reference for describing aberrations of the peripheral visual field.

Methods for aligning the eye during measurement.

Summary

The committee recommends that instruments designed to measure the optical properties of the eye and its aberrations be aligned co-axially with the eye's line-of-sight.



Fig. 3. Left panel illustrates the visual axis and panel right illustrates the line of sight.

Background

There are numerous ways to align the line of sight to the optical axis of the measuring instrument. Here we present simple examples of an objective method and a subjective method to achieve proper alignment.

Objective method

In the objective alignment method schematically diagramed in Fig. 4, the experimenter aligns the subject's eye (which is fixating a small distant target on the optical axis of the measurement system) to the measurement system. Alignment is achieved by centering the subject's pupil (by adjusting a bite bar) on an alignment ring (e.g., an adjustable diameter circle) which is co-axial with the optical axis of the measurement system. This strategy forces the optical axis of the measurement device to pass through the center of the entrance pupil. Since the fixation target is on the optical axis of the measurement device, once the entrance pupil is centered with respect to the alignment ring, the line-of-sight is co-axial with the optical axis of the measurement system.



Fig. 4. Schematic of a generic objective alignment system designed to place the line of sight on the optical axis of the measurement system. BS: beam splitter, FP: on axis fixation point.

Subjective method

In the subjective alignment method schematically diagramed in Figure 5, the subject adjusts the position of their own pupil (using a bite bar) until two alignment fixation points at different optical distances along and co-axial to the optical axis of the measurement device are superimposed (similar to aligning the sights on rifle to a target). Note that one or both of the alignment targets will be defocused on the retina. Thus the subject's task is to align the centers of the blur circles. Assuming the chief ray defines the centers of the blur circles for each fixation point, this strategy forces the line of sight to be co-axial with the optical axis of the measurement system. In a system with significant amounts of asymmetric aberration (e.g., coma), the chief ray may not define the center of the blur circle. In practice, it can be useful to use the subjective strategy for preliminary alignment and the objective method for final alignment.



Fig. 5. Schematic of a generic subjective alignment system designed to place the line of sight on the optical axis of the measurement system. BS: beam splitter, FP: fixation point source.

Conversion between reference axes

If optical aberration measurements are made with respect to some other reference axis, the data must be converted to the standard reference axis (see the tools developed by Susana Marcos at our temporary web site: //color.eri.harvard/standardization). However, since such conversions involve measurement and/or estimation errors for two reference axes (the alignment error of the measurement and the error in estimating the new reference axis), it is preferable to have the measurement axis be the same as the line-of-sight.

Description of Zernike Polynomials

The Zernike polynomials are a set of functions that are orthogonal over the unit circle. They are useful for describing the shape of an aberrated wavefront in the pupil of an optical system. Several different normalization and numbering schemes for these polynomials are in common use. Below we describe the different schemes and make recommendations towards developing a standard for presenting Zernike data as it relates to aberration theory of the eye.

Double Indexing Scheme

The Zernike polynomials are usually defined in polar coordinates (,), where is the radial coordinate ranging from 0 to 1 and is the azimuthal component ranging from 0 to 2. Each of the Zernike polynomials consists of three components: a normalization factor, a radial-dependent component and an azimuthal-dependent component. The radial component is a polynomial, whereas the azimuthal component is sinusoidal. A double indexing scheme is useful for unambiguously describing these functions, with the index *n* describing the highest power (order) of the radial polynomial and the index *m* describing the azimuthal frequency of the sinusoidal component. By this scheme the Zernike polynomials are defined as

$$Z_n^m(\rho,\theta) = \frac{N_n^m R_n^{m}(\rho) \cos m\theta \; ; \; for \; m \quad 0}{-N_n^m R_n^{m}(\rho) \sin m\theta \; ; \; for \; m < 0}$$
(1)

where N_n^m is the normalization factor described in more detail below and $R_n^{|m|}(\rho)$ is given by

$$R_{n}^{|m|}(\rho) = \frac{(n-|m|)/2}{s=0} \frac{(-1)^{s}(n-s)!}{s! \left[0.5(n+|m|-s]! \left[0.5(n-|m|-s]!\right]} \rho^{n-2s}\right]}$$
(2)

This definition uniquely describes the Zernike polynomials except for the normalization constant. The normalization is given by

$$N_{n}^{m} = \sqrt{\frac{2(n+1)}{1+\delta_{m0}}}$$
(3)

where $_{m0}$ is the Kronecker delta function (i.e. $_{m0} = 1$ for m = 0, and $_{m0} = 0$ for m = 0). Note that the value of *n* is a positive integer or zero. For a given *n*, *m* can only take on values -n, -n + 2, -n +4, ...n.

When describing individual Zernike terms, the two index scheme should always be used. Below are some examples.

Good:

"The values of $Z_3^{-1}(,)$ and $Z_4^2(,)$ are 0.041 and -0.121, respectively." "Comparing the astigmatism terms, $Z_2^{-2}(,)$ and $Z_2^2(,)$..."

Bad

"The values of $Z_7(\ ,\)$ and $Z_{12}(\ ,\)$ are 0.041 and -0.121, respectively." "Comparing the astigmatism terms, $Z_5(\ ,\)$ and $Z_6(\ ,\)$..."

Single Indexing Scheme

Occasionally, a single indexing scheme is useful for describing Zernike expansion coefficients. Since the polynomials actually depend on two parameters, n and m, ordering of a single indexing scheme is arbitrary. To avoid confusion, a standard single indexing scheme should be used, and this scheme should only be used for bar plots of expansion coefficients (Fig. 6). To obtain the single index, j, it is convenient to lay out the polynomials in a pyramid with row number n and column number m as shown in Table 1.

Table 1. Zernike pyramid. Row number is polynomial order n, column number is sinusoidal frequency m, table entry is the single-index j.



The single index, j, starts at the top of the pyramid and steps down from left to right. To convert between j and the values of n and m, the following relationships can be used:





Fig. 6. Example of a bar plot using the single index scheme for Zernike coefficients.



Fig. 7. Conventional right-handed coordinate system for the eye in Cartesian and polar forms.

Coordinate System

Typically, a right-handed coordinate system is used in scientific applications as shown in Fig. 7. For the eye, the coordinate origin is at the center of the eye's entrance pupil, the +x axis is horizontal pointing to the right, the +y axis is vertical pointing up, and the +z Cartesian axis points out of the eye and coincides with the foveal line-of-sight in object space, as defined by a chief ray emitted by a fixation spot. Also shown are conventional definitions of the polar coordinates $r = \sqrt{x^2 + y^2}$ and $= \tan^{-1}(y/x)$. This definition gives x = rcos and y = rsin. We note that Malacara [5] uses a polar coordinate system in which x = rsin and y = rcos. In other words, is measured clockwise from the +y axis (Figure 1b), instead of counterclockwise from the +x axis (Figure 1a). Malacara's definition stems from early (precomputer) aberration theory and is not recommended. In ophthalmic optics, angle is called the "meridian" and the same coordinate system applies to both eyes.

Because of the inaccessibility of the eye's image space, the aberration function of eyes are usually defined and measured in object space. For example, objective measures of ocular aberrations use light reflected out of the eye from a point source on the retina. Light reflected out of an aberration-free eye will form a plane-wave propagating in the positive z-direction and therefore the (x,y) plane serves as a natural reference surface. In this case the wavefront aberration function W(x,y) equals the z-coordinate of the reflected wavefront and may be interpreted as the shape of the reflected wavefront. By these conventions, W > 0 means the wavefront is phase-advanced relative to the chief ray. An example would be the wavefront reflected from a myopic eye, converging to the eye's far-point. A closely related quantity is the optical path-length difference (OPD) between a ray passing through the pupil at (x,y) and the chief ray point passing through the origin. In the case of a myopic eye, the path length is shorter for marginal rays than for the chief ray, so OPD < 0. Thus, by the recommended sign conventions, OPD(x,y) = -W(x,y).

Bilateral symmetry in the aberration structure of eyes would make W(x,y) for the left eye the same as W(-x,y) for the right eye. If W is expressed as a Zernike series, then bilateral symmetry would cause the Zernike coefficients for the two eyes to be of opposite sign for all those modes with odd symmetry about the y-axis (e.g. mode Z_2^{-2}). Thus, to facilitate direct comparison of the two eyes, a vector R of Zernike coefficients for the right eye can be converted to a symmetric vector L for the left eye by the linear transformation L=M*R, where M is a diagonal matrix with elements +1 (no sign change) or -1 (with sign change). For example, matrix M for Zernike vectors representing the first 4 orders (15 modes) would have the diagonal elements [+1, +1, -1, -1, +1, +1, +1, -1, -1, -1, -1, +1, +1, +1]

j =	n =	m =	$Z^{m}($,)
index	order	frequency	- <u>n</u> (,)
0	0	0	1
1	1	-1	2 sin
2	1	1	2 cos
3	2	-2	$\sqrt{6}$ ² sin 2
4	2	0	$\sqrt{3} (2^{-2}-1)$
5	2	2	$\sqrt{6}^{-2}\cos 2$
6	3	-3	$\sqrt{8}$ ³ sin 3
7	3	-1	$\sqrt{8} (3^{-3}-2) \sin(2)$
8	3	1	$\sqrt{8} (3^{-3}-2) \cos(3^{-3}-2) $
9	3	3	$\sqrt{8}$ ³ cos 3
10	4	-4	$\sqrt{10}$ ⁴ sin 4
11	4	-2	$\sqrt{10} (4^{4}-3^{2}) \sin 2$
12	4	0	$\sqrt{5} (6^{4}-6^{2}+1)$
13	4	2	$\sqrt{10} (4^{4}-3^{2}) \cos 2$
14	4	4	$\sqrt{10}$ ⁴ cos 4
15	5	-5	$\sqrt{12}$ ⁵ sin 5
16	5	-3	$\sqrt{12} (5^{-5} - 4^{-3}) \sin 3$
17	5	-1	$\sqrt{12} (10^{-5} - 12^{-3} + 3) \sin(10^{-5} - 12^{-3} + 3) \sin(10^{-5} - 12^{-3} + 3) \sin(10^{-5} - 12^{-5} + 3) \sin(10^{-5} - 12$
18	5	1	$\sqrt{12}$ (10 ⁻⁵ -12 ⁻³ +3) cos
19	5	3	$\sqrt{12} (5^{-5} - 4^{-3}) \cos 3$
20	5	5	$\sqrt{12}$ ⁵ cos 5
21	6	-6	$\sqrt{14}$ ⁶ sin 6
22	6	-4	$\sqrt{14} (6^{-6} - 5^{-4}) \sin 4$
23	6	-2	$\sqrt{14}$ (15 ⁶ -20 ⁴ +6 ²) sin 2
24	6	0	$\sqrt{7}$ (20 ⁶ -30 ⁴ +12 ² -1)
25	6	2	$\sqrt{14}$ (15 ⁶ -20 ⁴ +6 ²) cos 2
26	6	4	$\sqrt{14} (6^{-6} - 5^{-4}) \cos 4$
27	6	6	$\sqrt{14}$ ⁶ cos 6
28	7	-7	$4^{-7}\sin 7$
29	7	-5	$4 (7^{-7}-6^{-5}) \sin 5$
30	7	-3	4 $(21^{-7}-30^{-5}+10^{-3})\sin 3$
31	7	-1	4 $(35^{7}-60^{5}+30^{3}-4)$ sin

Table 2. Listing of Zernike Polynomials up to 7th order (36 terms)

32	7	1	4 $(35^{-7}-60^{-5}+30^{-3}-4)\cos$
33	7	3	4 $(21^{7}-30^{5}+10^{3})\cos 3$
34	7	5	4 $(7^{-7}-6^{-5})\cos 5$
35	7	7	$4^{-7}\cos 7$

A standard aberrator for calibration

The original goal was to design a device that could be passed around or mass-produced to calibrate aberrometers at various laboratories. We first thought of this as an aberrated model eye, but that later seemed too elaborate. One problem is that the subjective aberrometers needed a sensory retina in their model eye, while the objective ones needed a reflective retina of perhaps known reflectivity. We decided instead to design an aberrator that could be used with any current or future aberrometers, with whatever was the appropriate model eye.

The first effort was with a pair of lenses that nearly cancelled spherical power, but when displaced sideways would give a known aberration. That scheme worked, but was very sensitive to tilt, and required careful control of displacement. The second design was a trefoil phase plate (OPD = Z_3^3 = $r^3 \sin 3$) loaned by Ed Dowski of CDM Optics, Inc. This 3rd order aberration is similar to coma, but with three lobes instead of one, hence the common name "trefoil". Simulation of the aberration function for this plate in ZEMAX® is shown in Figs. 8, 9. Figure 8 is a graph of the Zernike coefficients showing a small amount of defocus and 3rd order spherical aberration, but primarily C_3^3 . Figure 9 shows the wavefront, only half a micron (one wave) peak to peak, but that value depends on , above.



Fig. 8: Zernike coefficients of trefoil phase plate from ZEMAX® model (note different numbering convention from that recommended above for eyes).



Fig. 9: Wavefront map for trefoil phase plate from the ZEMAX® model

We mounted the actual plate and found that it had even more useful qualities: As the phase plate is translated across the pupil, it adds some C_2^2 , horizontal astigmatism. When the plate is perfectly centered, that coefficient is zero. Further, the slope of C_2^2 (x) measures the actual pupil.

$$Z_3^3(x - x_0) = \kappa (r - x_0) \sin 3\theta = \kappa (3xy^2 - x^3)$$
⁽⁷⁾

so

$$\frac{Z_3^3(x-x_0)}{x} = \Im(\Im y^2 - x^2) = \Im Z_2^2$$
⁽⁸⁾

and similarly

$$\frac{Z_3^{-3}(x-x_0)}{x} = -6\kappa xy = -3Z_2^{-2}$$
(9)

This means that $Z_3^3 = 3Z_2^2 x$ and then, since $W = C_n^m Z_n^m$, we get a new term proportional to x. Plotting the coefficient C_2^2 against x, we need to normalize to the pupil size. That could be useful as a check on whether the aberrator is really at the pupil, or whether some smoothing has changed the real pupil size, as measured. Figures 10-13 confirm this behavior and the expected variation with rotation (3).



Fig. 10: Wavefront map from the aberrator, using the SRR aberrometer.



Albitrary Holizontal Fosition (pupil radius dints,

Fig. 12: Zerike coefficients are stable against horizontal displacement, except for C_3^{-3} .



Fig. 11: The phase plate of Figure 10 has been moved horizontally 4 mm.



Fig. 13: Zernike coefficients C_3^{-3} and C_3^{-3} as a function of rotation of the phase plate about the optic axis.

Although the phase plate aberrator works independently of position in a collimated beam, some aberrometers may want to use a converging or diverging beam. Then it should be placed in a pupil conjugate plane. We have not yet built the mount for the phase plate, and would appreciate suggestions for that. Probably we need a simple barrel mount that fits into standard lens holders – say 30 mm outside diameter. We expect to use a standard pupil, but the phase plate(s) should have 10 mm clear aperture before restriction. The workshop seemed to feel that a standard pupil should be chosen. Should that be 7.5 mm?

We have tested the Z_3^3 aberrator, but it may be a good idea to have a few others. We borrowed this one, and it is somewhat fragile. Bill Plummer of Polaroid thinks he could generate this and other plates in plastic for "a few thousand dollars" for each design. Please send suggestions as to whether other designs

are advisable (webb@helix.mgh.Harvard.edu), and as to whether we will want to stack them or use them independently. That has some implications for the mount design, but not severe ones. We suggest two Z_3^3 plates like this one, and perhaps a Z_6^0 , fifth order spherical.

At this time, then, our intent is to have one or more standard aberrators that can be inserted into any aberrometer. When centered, and with a standard pupil, all aberrometers should report the same Zernike coefficients. We do not intend to include positioners in the mount, assuming that will be different for each aberrometer.

Another parameter of the design is the value of . That comes from the actual physical thickness and the index of refraction. Suggestions are welcome here, but we assume we want coefficients that are robust compared to a diopter or so of defocus.

The index will be whatever it will be. We will report it, but again any chromaticity will depend on how it's used. We suggest that we report the expected coefficients at a few standard wavelengths and leave interpolation to users.

Plans for Phase II

Reference Axes Sub-committee

- develop a shareware library of software tools needed to convert data from one ocular reference axis to another (e.g., convert a wavefront aberration for the corneal surface measured by topography along the instrument's optical axis into a wavefront aberration specified in the eye's exit pupil plane along the eye's fixation axis.)
- generate test datasets for evaluating software tools

Describing functions subcommittee

- develop a shareware library of software tools for generating, manipulating, evaluating, etc. the recommended describing functions for wavefront aberrations and pupil apodizing functions.
- develop additional software tools for converting results between describing functions (e.g. converting Taylor polynomials to Zernike polynomials, or converting single-index Zernikes to double-index Zernikes, etc.)
- generate test datasets for evaluating software tools

Model eyes subcommittee

- build a physical model eye that can be used to calibrate experimental apparatus for measuring the aberrations of eyes
- circulate the physical model to all interested parties for evaluation, with results to be presented for discussion at a future VSIA meeting.

Acknowledgements

The authors wish to thank the numerous committee members who contributed to this project.

References

- L. N. Thibos, R. A. Applegate, H. C. Howland, D. R. Williams, P. Artal, R. Navarro, M. C. Campbell, J. E. Greivenkamp, J. T. Schwiegerling, S. A. Burns, D. A. Atchison, G. Smith, and E. J. Sarver, "A VSIA-sponsored effort to develop methods and standards for the comparison of the wavefront aberration structure of the eye between devices and laboratories.," in *Vision Science and Its Applications*, (Optical Society of America, Washington, D.C., 1999) pp 236-239.
- 2. L. N. Thibos, A. Bradley, D. L. Still, X. Zhang, and P. A. Howarth, "Theory and measurement of ocular chromatic aberration," Vision Research **30**, 33-49 (1990).
- 3. A. Bradley and L. N. Thibos (Presentation 5) at http://www.opt.indiana.edu/lthibos/ABLNTOSA95
- 4. A. G. Bennett and R. B. Rabbetts, Clinical Visual Optics, 2nd ed., (Butterworth, 1989).
- 5. D. Malacara, Optical Shop Testing, 2nd ed., (John Wiley & Sons, Inc., New York, 1992).