Diffraction

Diffraction describes the tendency for light to bend around corners.

Huygens principle

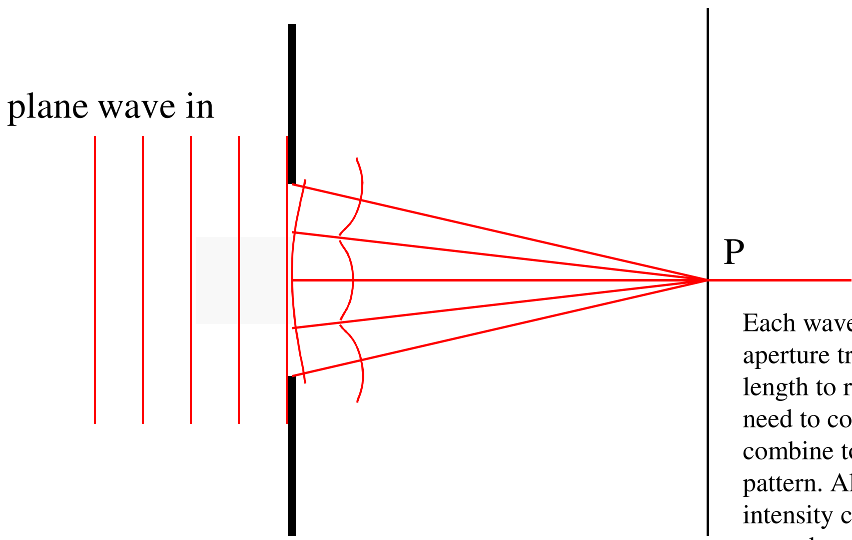
All points on a wavefront can be considered as point sources for the production of secondary wavelets, and at a later time the new wavefront position is the envelope (or surface of tangency) to these secondary wavelets.

Fresnel's Addition

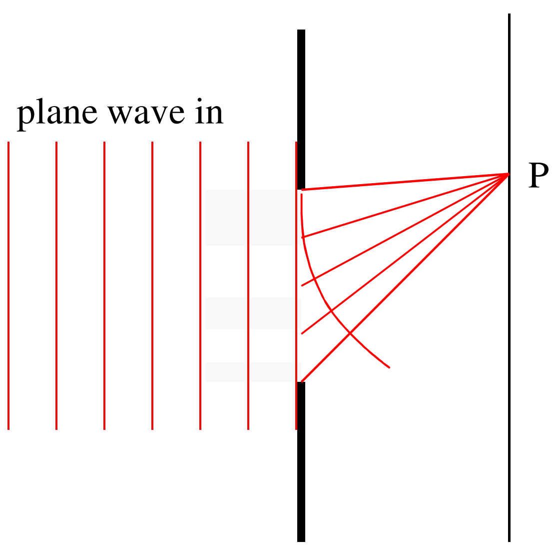
For light in the same bundle (ie. waves from the same point source) all waves are mutually coherent and all interfere with each other.

Fresnel's diffraction occurs in the near field when the screen is held close to the aperture.

Fraunhofer diffraction occurs in the far field or near the focal plane created by converging the light with a positive lens. These diffraction effects are relevant for imaging systems like the eye, telescopes, cameras etc.

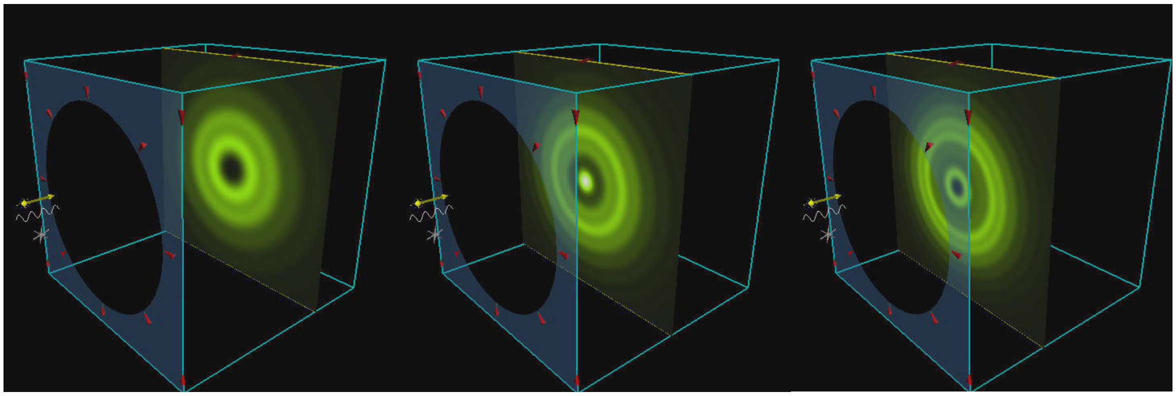


Each wave of light across the aperture travels a different path length to reach the screen. You need to consider how all waves combine to generate the diffraction pattern. Along the axis, the intensity could be high, or could be zero, depending on the distance.



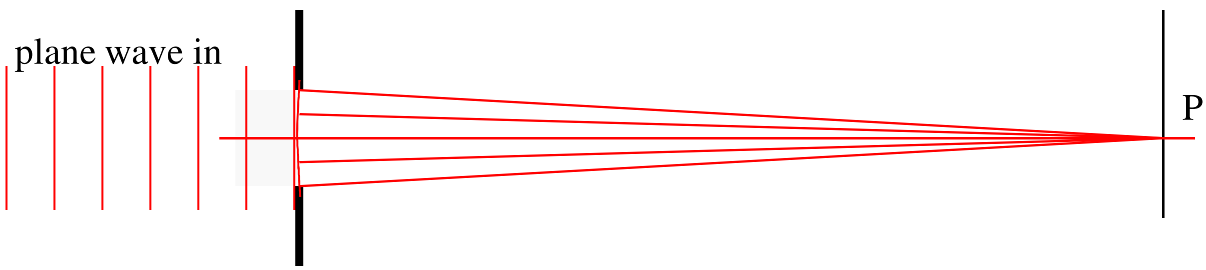
Away from the axis, we again have to add all the amplitudes across the wavefront to get an estimate of whether the diffraction pattern has a peak or a valley. When you go very far off the axis, there is an equal amount of in and out of phase waves so the intensity drops to zero.

The patterns that you observe across the screen are called **Fresnel Diffraction Patterns.** These are observed when the screen is held near to the aperture.

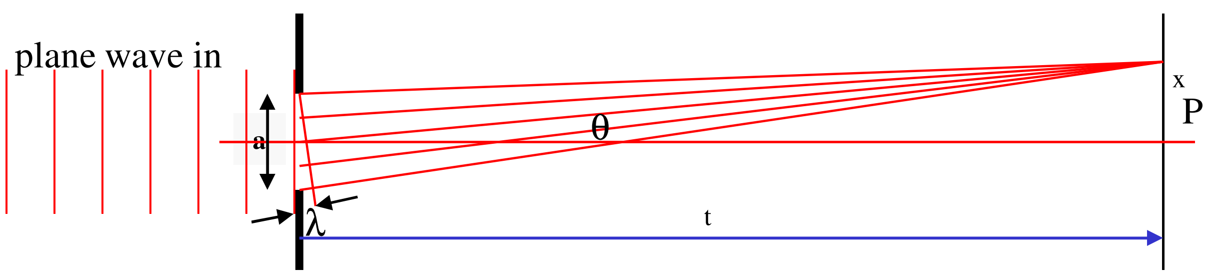


In thse figures the aperture is on the right and the screen is on the left. You can see how the intensity distribution on the screen will vary as the screen is moved toward an aperture. (images from **David C. Banks, John T Foley, Kiril N. Vidimce and Ming-Hoe Kiu, "Instructional Software for Visualizing Optical Phenomena," 1997 IEEE Visualization Conference.)**

Now we move very far away from the aperture. The effect is that the distance for each ray across the aperture to optical axis at the screen is very nearly the same. This called the **far field**. In the far field, there is always a peak at the center because all the light is in phase



Now if we look off the axis, we introduce a phase difference across the rays. When the phase difference at the edges is equal to 1 wavelength. One half of the waves are out of phase with the other half and we get the first low intensity point in the diffraction pattern. The diffraction pattern is called the **Fraunhofer Diffraction Pattern**.Note that this is opposite to the case of the double slit, because now we have to include the phase of all the rays.



In this diffraction pattern, for small angles, the positions of the **minima** occur at: asin(θ) = mλ,

where m =± 1,± 2….and a is the width of the slit. For a screen at a distance s,

$$\sin\theta \approx \theta \approx \frac{x}{t} \quad \therefore \quad \text{minima occur when} \quad x = \frac{m\lambda t}{d} \quad m = \pm 1, \pm 2 ...$$

note that the equation is the same as for the double slit, except it is used to locate the minima, and there is no m=zero

Example (example 22.1a from Keating p 498)

A slit of width 0.5 mm is illuminated with 633 nm light. At what angular location is the first minimum observed in the single slit diffraction pattern on a distant screen?

$$0.5\times10^{-3}\sin\theta = 633\times10^{-9}\cdot m$$

$$\sin\theta = \frac{633\times10^{-9}}{0.5\times10^{-3}} = 1.27\times10^{-3}$$

using a small angle approximation

$$\theta = 1.27\times10^{-3} \text{ radians}$$

Where is the location of the minimum if the screen is 10 m away from the slit?

$$\theta = 1.27\times10^{-3} \text{ radians}$$

$$x = 10\cdot\tan\theta \cong 10\cdot\theta = 1.27 \text{ cm}$$

Intensity profile of the diffraction pattern

The central peak is the most intense of all. This is because all of the light is in phase at that point. As you move further off-axis, the presence of out of phase zones reduce the amplitude and also there is an overall fall-off in intensity due to the fact that you are getting further from the slits. An infinitely small slit will produce a uniform illumination with the expected fall-off due to distance from the slit. The amplitude and intensity profiles are given by:

$$E = \frac{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}{\left(\frac{\pi d\sin\theta}{\lambda}\right)} \qquad \Rightarrow \qquad I = E^2 = \left(\frac{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}{\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right)^2$$

this form, $\frac{\sin(x)}{x}$, is called the sinc function

The zeros occur when:

$$\frac{\pi d\sin\theta}{\lambda} = m\cdot\pi, \qquad \text{where } m = \pm1, \pm2...$$

$$\sin\theta = \frac{m\pi\lambda}{\pi d} = \frac{m\lambda}{d}$$

for small angles,

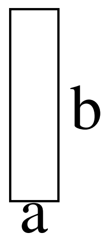
$$\sin\theta \cong \frac{x}{t} \qquad \Rightarrow \qquad x = \frac{m\lambda t}{d}$$

The diffraction pattern is spread perpendicular to the direction of the slit (ie if the slit is vertical, the diffraction pattern is horizontal).

Fraunhofer diffraction patterns for other apertures.

Rectangular aperture:

A square aperture can be considered as two slit apertures in the vertical and horizontal directions. The *sinc* functions for each slit aperture are simply multiplied together.



$$E = \frac{\sin(\alpha)}{\alpha} \cdot \frac{\sin(\beta)}{\beta}, \quad \text{where } \alpha = \frac{\pi a \sin\theta}{\lambda}, \beta = \frac{\pi b \sin\phi}{\lambda}$$

Circular aperture:

This is the most relevant and important aperture shape since it is the shape of most lenses and apertures. The diffraction pattern for a circular aperture is defined as:

$$E = \frac{2 \cdot J_1\left(\frac{\pi d \sin\theta}{\lambda}\right)}{\left(\frac{\pi d \sin\theta}{\lambda}\right)} \qquad \Rightarrow \qquad I = E^2 = \left(\frac{2 \cdot J_1\left(\frac{\pi d \sin\theta}{\lambda}\right)}{\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right)^2$$

Where J_1 is the Bessel function. The shape that this equation defines is called the Airy disc. The radius to the first minimum in this function is:

$$\theta = \frac{1.22 \cdot \lambda}{d} \qquad \text{also, since } \theta \cong x/t$$

$$x = \frac{1.22 \cdot \lambda t}{d} \qquad \Leftarrow \quad \text{compare with} \quad x = \frac{\lambda t}{d} \text{ for first minimum through a slit aperture}$$

I have been stating that the Fraunhofer diffraction pattern can be obtained by moving the screen far from the aperture. A Fraunhofer diffraction pattern can also be generated at the focal point of a lens. So Fraunhofer diffraction patterns are also observed in the image plane. In order to generate a pattern, you set the lens against the aperture and simply use the focal length of the lens as the aperture to screen distance (ie replace s with f')
Everything else is the same.

Diffraction is another factor that limits the image quality in optical systems. Recall that the size of the diffraction pattern is inversely proportional to the diameter of the aperture. Therefore, in a lens, the bigger the aperture, the smaller the Airy disc.

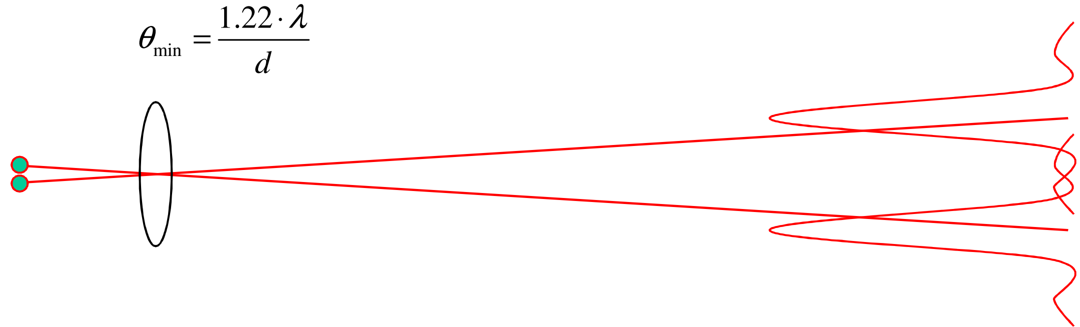
Two Point Resolution

Consider two **mutually incoherent** point sources. When the sources are mutually incoherent, you simply add the intensities. Each source produces an Airy disc in the image plane. When the sources get close together, the two Airy disc patterns begin to overlap until they can no longer be resolved as distinct objects.

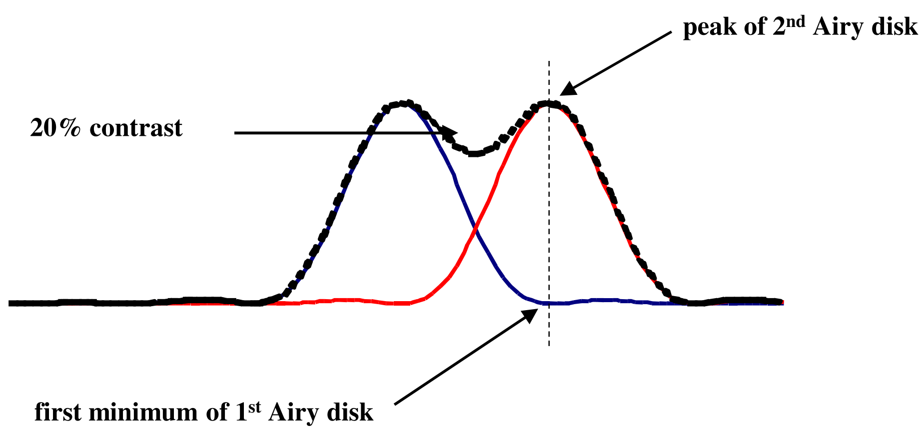
The smallest value of the angle for which images of two points can be detected as two distinct objects is the limit of resolution. We have to adopt an arbitrary standard to come up with a number for the resolution limit so we define the limit as when the peak from one Airy disc sits atop the first minimum of the other distribution. This is called the **Rayleigh resolution limit**. The equation that defines this condition is simply the distance from the center to the first minimum which I showed earlier:

$$x_{\min} = \frac{1.22 \cdot \lambda t}{d} \qquad \text{also, since } \theta \cong x/t$$

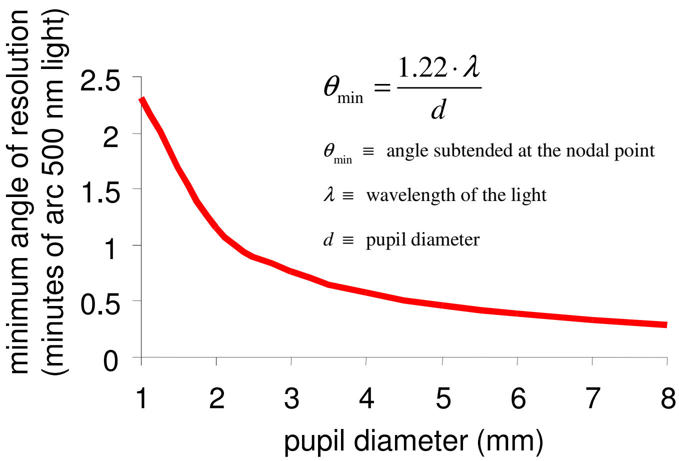
$$\theta_{\min} = \frac{1.22 \cdot \lambda}{d}$$



This shows the images of two point sources that are considered to be resolved at the Rayleigh resolution limit



Recall that aberrations tend to blur ideal images. But even if there were no aberrations, rays would not be imaged to a point source but to a diffraction pattern. Perfect optical systems that have no aberrations are still limited by diffraction. A system that is free from aberrations is called **diffraction-limited** (ie its image quality is limited solely by diffraction)



The graph plots the minimum angle separation that two point objects have to be separated by to be resolved in a diffraction-limited system. The smaller the minimum angle of resolution, the better the vision.

Example.

A diffraction-limited eye with a 6 mm pupil is looking at an approaching car whose headlights have a wavelength of 550 nm and are separated by 1.5 m.

What is the minimum angular resolution of the eye?

At what distance can you no longer resolve two distinct headlights?

$$\theta_{min} = \frac{1.22 \cdot \lambda}{d} = \frac{1.22 \cdot 550 \times 10^{-9}}{6 \times 10^{-3}} = 0.00011183 \text{ radians } = 0.0064 \text{ deg } = 0.384 \text{ minutes of arc}$$

$$x_{min} = 1.5 \text{ m} = \frac{1.22 \cdot 550 \times 10^{-9} \cdot t}{6 \times 10^{-3}} \quad \Rightarrow \quad t = 13{,}412.8 \text{ m } = 13.412 \text{ km} = 8.4 \text{ miles}$$

Two point resolution

Image quality in the eye:

For 630 nm light

6 mm pupil, angular resolution is about 0.44 minutes of arc.

3 mm pupil, angular resolution is about 0.881 minutes of arc.

How does this relate to vision?

The 20/20 letters on a Snellen Chart subtend 5 minutes of arc. The center lines on the two arms of the E are separated by 2 minutes of arc on the retina. The gap between two arms on the E is one minute of arc. The definition of what resolution is required to resolve an E is not clear cut, but you should expect to be able to resolve 20/20 letters if you are well-corrected. 20/10 letters are half the size and resolving them may be difficult for pupil sizes smaller than 3 mm.

Recall that aberrations reduce image quality when the pupil gets larger, so even though diffraction theory tells us that the image should get sharper, it does not.

If the paraxial optics are free from aberrations, why don't we just use a tiny pupil? Diffraction takes over for small pupils and makes the image quality worse. We are beaten at both ends of our pupil range. Given the aberrations that are present in the eye, the best point spread function is obtained for a pupil size of about 3 mm.