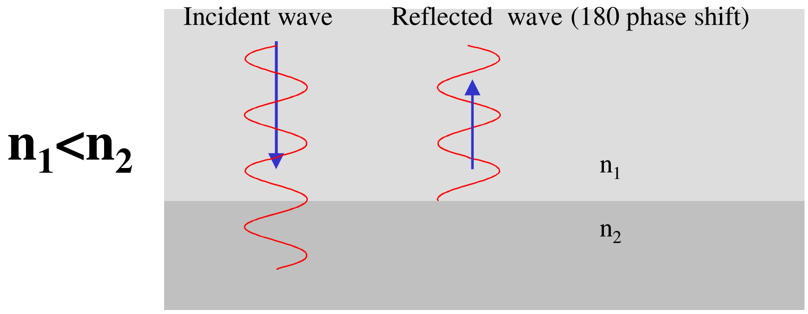
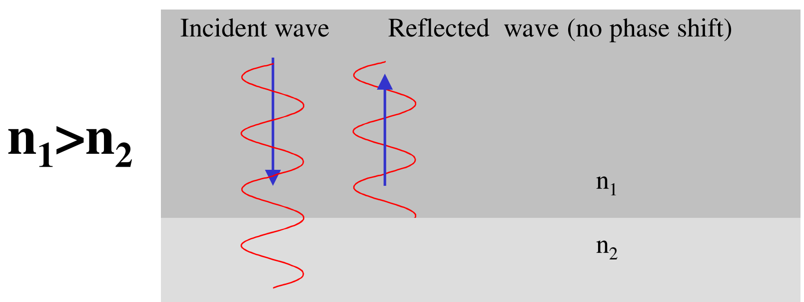
Thin Film Interference

When light hits a surface, it can be absorbed, transmitted or reflected.

Light can reflect off any interface where there is a change in refractive index. The rule for reflection is that when the index of refraction after the interface is higher then the index of refraction before the interface then the light, upon reflection, undergoes a 180 degree phase shift. When the index of refraction after the interface is less than the index before the interface, then there is no phase shift in the reflected light.

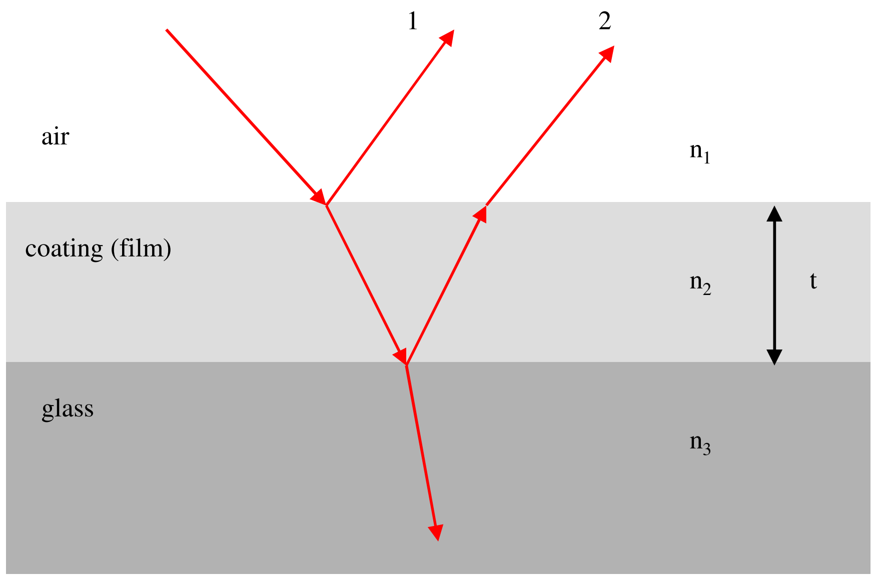




When light hits a material that has multiple layers, each layer can reflect light. The light reflected from any one of these layers might interfere with the light that has reflected off any of the other layers. Each layer creates its own image of a point source. These sources will be mutually coherent because they all arise from a single original source.

The condition for interference is that the difference in distance between the sources of light (from the reflections) be within the coherence length of the light.

<u>Diversion:</u> **Coherence Length**. Defined as the length in space over which the light has a predictable phase. A laser, for example, has a long coherence length. Although there are random fluctuations in phase over time they occur after the waves have traveled some meters in distance. A incandescent bulb on the other hand, has tremendously fast variations so the phase is predictable over only a very short period of space. Consider the double slit experiment: The interference arises from the difference in path lengths between the two mutually coherence sources of light. For high angles, the path difference can be come very large (multiple wavelengths). For a laser, which has a long coherence length, the two beams will still retain a fixed phase difference between them. For a normal light source, there will be so much difference in distance between the two waves that there will have been time for random phase jumps to occur. So the two sources are essentially mutually incoherent. The path difference must be within the coherence length for interference to occur. For a laser, the coherence length is on the order of meters. For an incandescent light bulb, the coherence length is only a few micrometers. For the sun, the coherence length is on the order of millimeters. For some lasers, the coherence length can be many kilometers.



$n_1<n_2<n_3$

The rays labeled 1 and 2 can interfere with each other. **Because each interface is at a transition from a lower to a higher refractive index, there is 180 deg phase shift at each reflection (for 1 and 2). So they undergo the same phase shift and we can therefore ignore it in this example.** All we need to do is calculate the phase difference between rays 1 and 2 due to the path length difference. If there is destructive interference, then there will be a minimum in the amount of reflected light (this is the design of an antireflection coating). If light constructively interferes, there will be an increase in the amount of reflection (this is the design of highly reflective mirror). Since light has to go somewhere, the light that is not reflected will have to be transmitted, so both transmission and reflection are affected by thin film coatings.

The difference in actual path length between 1 and 2 is (assuming that the angles are small):

$$\Delta path_{1,2} = 2t$$

The difference in the number of waves between 1 and 2 is

$$\Delta waves_{1,2} = \frac{2t}{\lambda / n_2} = \frac{2tn_2}{\lambda}$$

For destructive interference to occur, we require that rays 1 and 2 be 180 degrees out of phase, or that the number of waves of difference be 1/2

$$\frac{1}{2} = \frac{2tn_2}{\lambda} \qquad \Rightarrow \qquad t_{dest} = \tfrac{1}{4}\frac{\lambda}{n_2}$$

Similar interference can occur when the difference in the number of waves is 1.5, 2.5 3.5 ….

$$t_{dest} = \tfrac{1}{4}\frac{\lambda}{n_2}, \tfrac{3}{4}\frac{\lambda}{n_2}, \tfrac{5}{4}\frac{\lambda}{n_2}, \tfrac{7}{4}\frac{\lambda}{n_2}..... \quad t = \frac{\left(m+\tfrac{1}{2}\right)}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2...$$

But recall the requirement that interference can only occur when the difference in path length of two mutually coherent sources of light be within the coherence length. So as the thickness increases, many sources will no longer be mutually coherent.

The same holds for constructive interference. Constructive interference will occur when the difference in waves between rays 1 and 2 is some integer multiple of wavelengths.

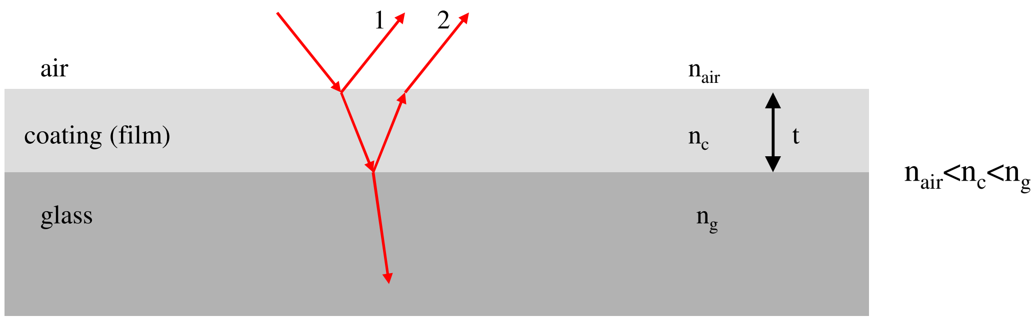
$$1 = \frac{2tn_2}{\lambda} \quad \Rightarrow \quad t_{const} = \tfrac{1}{2}\frac{\lambda}{n_2}, 1\frac{\lambda}{n_2}, \tfrac{3}{2}\frac{\lambda}{n_2}, 2\frac{\lambda}{n_2}... \quad t_{const} = \frac{m}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2...$$

So, imagine that you could vary the thickness of a coating on a piece of glass, the amount of reflection would vary as the thickness requirements for constructive and destructive interference were met. Manipulating the coatings on surfaces can be used for many applications, which I will talk about in the next couple of lectures.

Anti Reflection Coatings ARCs

To make an antireflection coating, or ARC, you put a coating on the surface of your glasses with the appropriate thickness that minimizes the reflection. By minimizing the reflection, you automatically increase the transmission.

Lens without antireflection coatings	Lens with ARC
4% off the first surface	0.5% off first surface
4% off the second surface	0.5 % off second surface
92% transmittance.	99% transmittance



But there is a second condition that needs to be satisfied in order to maximize the amount of destructive interference. For complete destructive interference, we also require that the *amplitudes* of ray 1 and 2 be the same. To meet this requirement we need to have the condition that the reflectivity of the air coating interface and the coating glass interface be the same.

Recall the reflectance equation for amplitude: $r = \frac{n' - n}{n' + n}$ for intensity: $R = (r)^2 = \left[\frac{n' - n}{n' + n}\right]^2$

For ray 1: $r_1 = \frac{n_c - n_{air}}{n_c + n_{air}}$ For ray 2: $r_2 = \frac{n_g - n_c}{n_g + n_c}$

For light that is out of phase, the resultant amplitude is the difference between the two amplitudes so total destructive interference occurs when

$$r_1 - r_2 = 0$$

$$\therefore \frac{n_c - n_{air}}{n_c + n_{air}} = \frac{n_g - n_c}{n_g + n_c}$$

yields the result:

$$n_c = \sqrt{n_g}$$

> For non-ideal refractive indices, the intensity is the square of the resultant non-zero amplitude.

To summarize, an effective ARC must meet the following two conditions:

Path condition (destructive interference): $t_{dest} = \frac{1}{4}\frac{\lambda}{n_c}$

Amplitude condition: $n_c = \sqrt{n_g}$

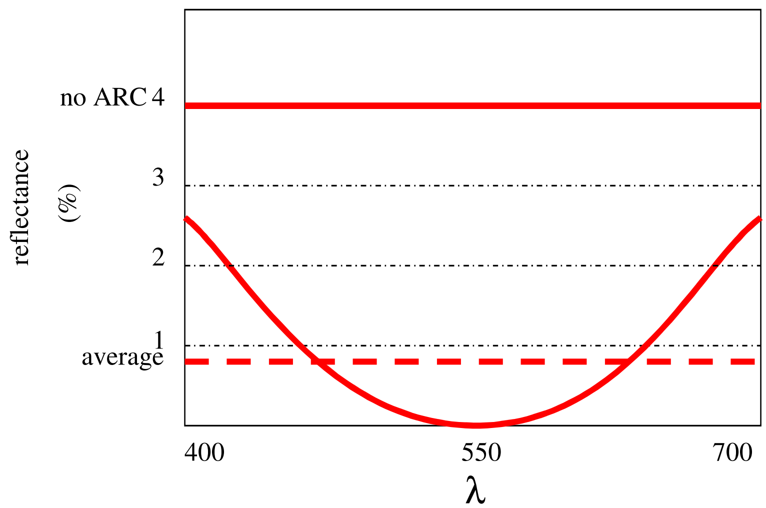
So, what are the problems with ARCs? They can make coatings of exact thicknesses, but the constraint is on available materials that meet the amplitude condition requirement.

Problem 1:

If we start with glass of index 1.5, we are forced to find a suitable coating material that has an index 1.22. Such materials are not readily available so manufacturers simply try to get as close as possible. They typically use a coating made of $MgFl_2$, which has an index of 1.38. The ideal glass for that coating would have an index of 1.9, which is not available. So the higher the index of the glass, the better the ARC, when you are using $MgFl_2$ as a coating.

Problem 2:

Wavelength dependency. The path condition can only be exactly met for one wavelength. Ideally, we choose to optimize the coating for wavelengths in the middle of the spectrum. So while 550 does not reflect, the lower and higher wavelengths do, which gives the reflection from ARC glasses a purplish hue.



A typical ARC has an average of about 2% reflectance

Problem #3:

Only works as designed on axis. When you look at the reflection off axis, it changes color because the path lengths through the coating are increased. It becomes a much more complicated problem

Example:

What is the reflectance of a glass (n=1.5) surface with a $MgFl_2$ coating (n=1.38) optimized for 550 nm light for

550 nm light?

400 nm light?

Solution:

Step 1: What is the thickness of the coating?

$$t_{dest} = \frac{1}{4}\frac{\lambda}{n_c} = \frac{1}{4}\frac{550}{1.38} = 99.64 \text{ nm}$$

Step 2: What is the amplitude of reflectance at the surfaces?

$$r_1 = \frac{n_c - n_{air}}{n_c + n_{air}} = \frac{1.38-1}{1.38+1} = 0.16 \qquad r_2 = \frac{n_g - n_c}{n_g + n_c} = \frac{1.5-1.38}{1.5+1.38} = 0.0417$$

Step 3: For 550 nm light….

$$I_{coherent} = (E_1 + E_2)^2 = A_1^2 + A_2^2 + 2\cdot A_1 A_2 \cos(p_1 - p_2)$$

$$p_1 - p_2 = 180 \text{ since they are out of phase}$$

$$I_{coherent} = A_1^2 + A_2^2 + 2\cdot A_1 A_2 \times (-1) = 0.159^2 + 0.0417^2 - 2\times 0.159 \times 0.0417$$

$$= 0.0138 \equiv 1.38\%$$

Step 4: For 400 nm light, what is the phase difference?

$$\Delta waves = \frac{2\times 99.64}{400/1.38} = 0.687 \text{ waves}$$

$$\Rightarrow \Delta phase = 0.687 \times 2 \times \pi = 4.32 \text{ radians}$$

Step 5: For 400 nm light…

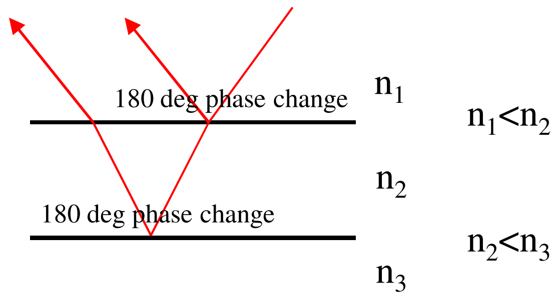
$$I_{coherent} = (E_1 + E_2)^2 = A_1^2 + A_2^2 + 2\cdot A_1 A_2 \cos(p_1 - p_2)$$

$$I_{coherent} = 0.159^2 + 0.0417^2 + 2\times 0.159\times 0.0417 \times \cos(4.32)$$

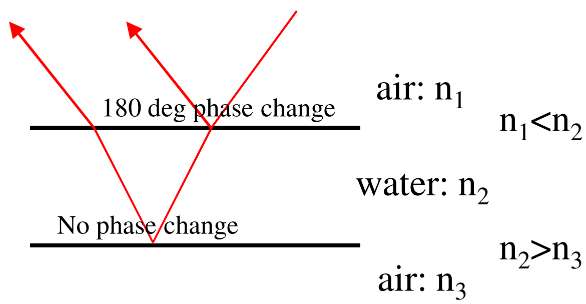
$$= 0.0219 \equiv 2.19\%$$

400 nm light is more reflective than 550 nm light, as expected. Reflectance was not zero for 550 nm light because the amplitude condition was not met.

Anti-Reflection Coating



Soap bubble film

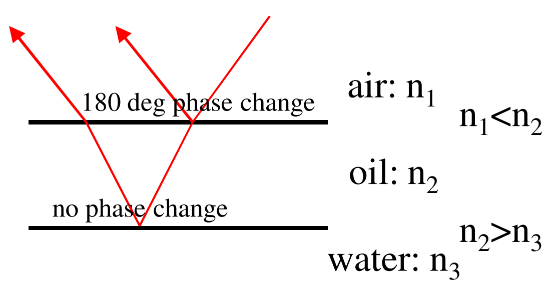


So the condition is opposite to that of an anti-reflection coating, because there is already a 180 deg phase difference between the two waves because one of the phases is not shifted. So, if the total path through the film is one half of a wavelength, then the light constructively interferes, not destructively as was the case for the ARC. Because the thickness of a soapy film constantly changes, you'll see a spectrum of different colors across the soap bubble.

What happens right before the bubble bursts? The layer becomes very very thin so that the thickness is nearly zero. At this point there is no significant path difference. The only thing changing the phase between the two rays is the 180 phase change from the reflection off the first surface. Since there is a 180 phase change, there is a minimum in reflectivity. This holds for all wavelengths, and so the bubble has no reflection at the moment before it breaks.

Oil film (like a soap bubble)

> The typical index of refraction of oil is about 1.5, which is similar to glass.



The thickness of the oil layer varies as well as your viewing angle, so you see a spectrum of colors. Like the soap bubble, the oil has low reflectance when the layer is very thin. That is why the reflection from an oily surface on a puddle has a blackish appearance.

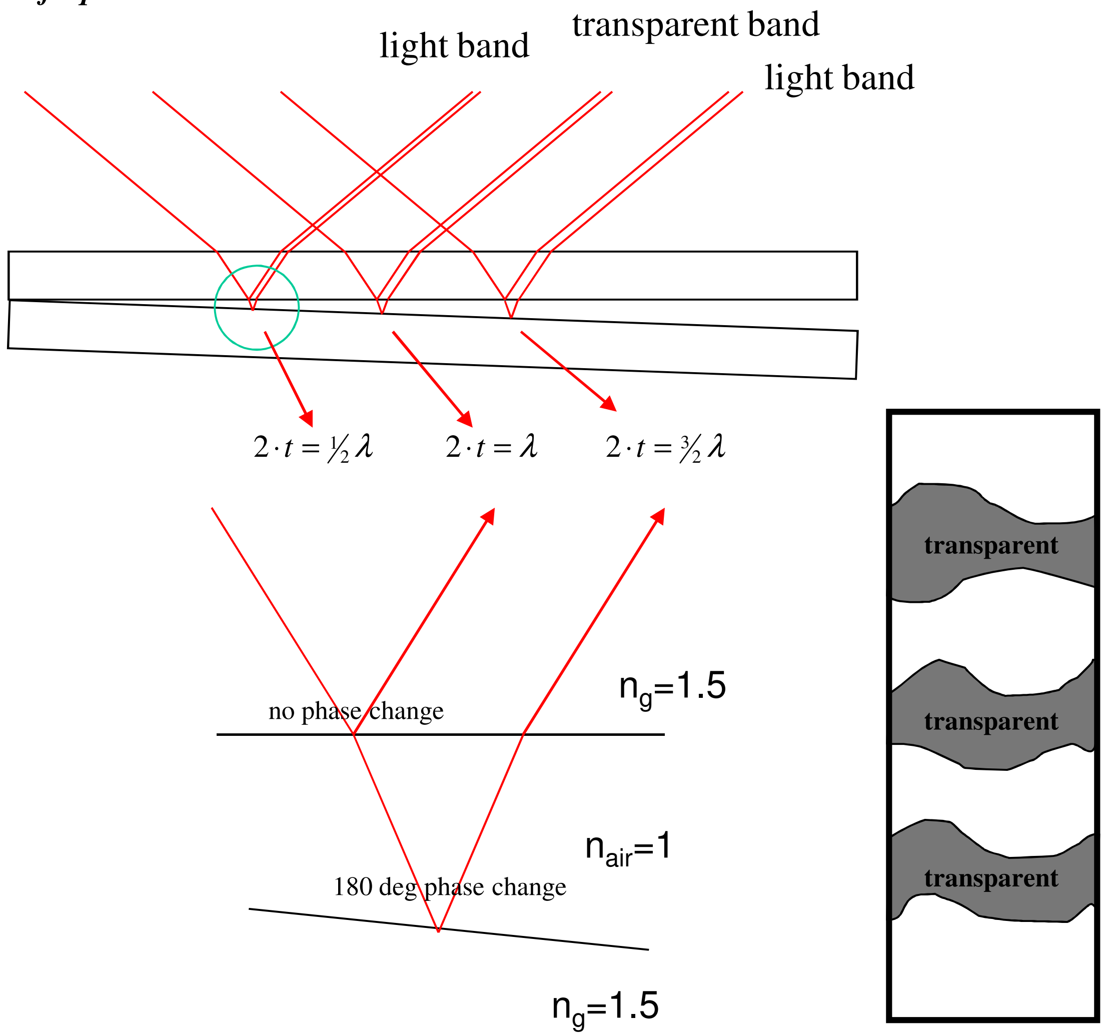
If the phase changes are **common** to both surfaces (eg ARC), then

$$t_{dest} = \frac{\left(m + \frac{1}{2}\right)}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2... \qquad t_{const} = \frac{m}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2...$$

If the phase changes are **not common** to both surfaces (eg soap bubble, or oil), then

$$t_{dest} = \frac{m}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2... \qquad t_{const} = \frac{\left(m + \frac{1}{2}\right)}{2}\frac{\lambda}{n_2}, \quad m = 0,1,2...$$

Fringes of equal thickness



When t = 1/4 λ you get constructive interference, when t = 1/2 λ you get destructive interference. In this case, the film is air so the wavelength need not be adjusted.

path condition for constructive interference

$$2 \cdot n_c t = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \ldots \quad \text{but } n_c = 1$$

$$2 \cdot t = \left(m + \tfrac{1}{2}\right)\lambda, \qquad m = 0, 1, 2.....$$

$$\text{or} \quad t_{const} = \frac{\left(m + \frac{1}{2}\right)\lambda}{2}, \qquad m = 0, 1, 2.....$$

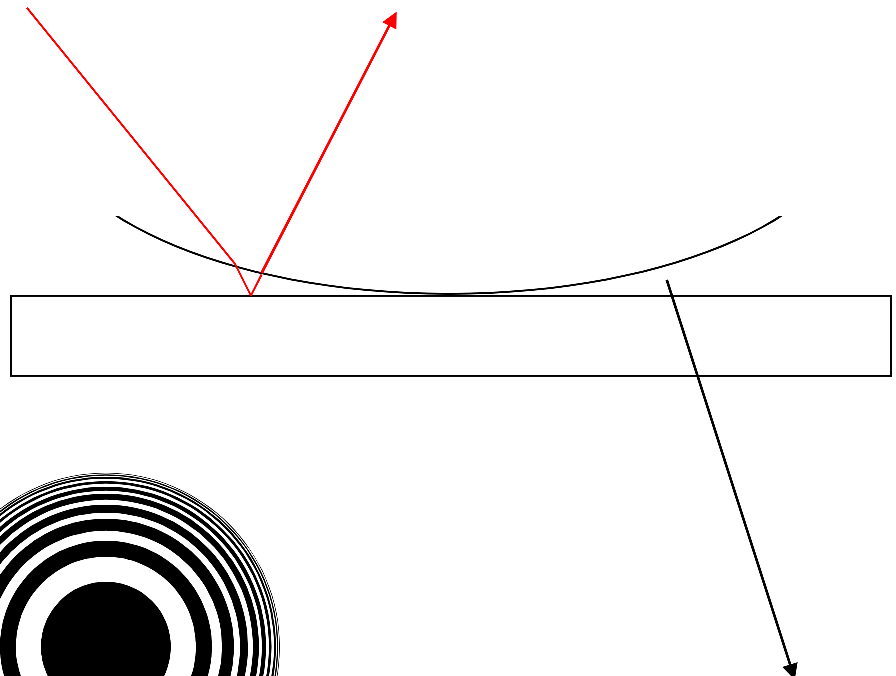
path condition for destructive interference

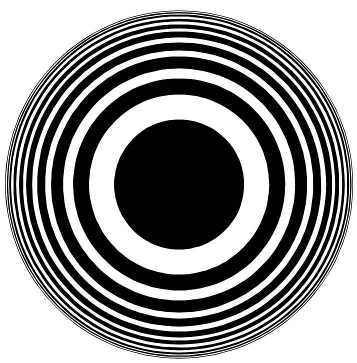
$$2 \cdot n_c t = 0, \lambda, 2\lambda, \ldots \quad \text{but } n_c = 1$$

$$2 \cdot t = (m)\lambda, \qquad m = 0, 1, 2.....$$

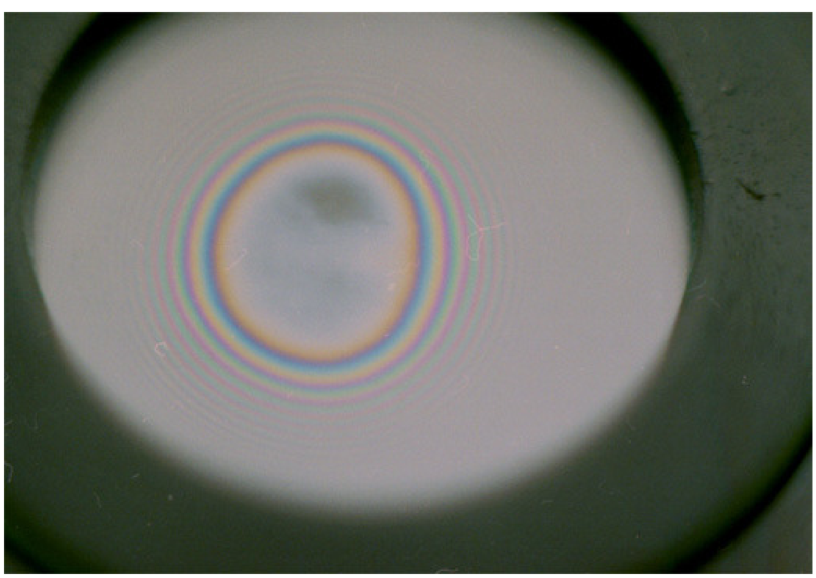
$$\text{or} \quad t_{dest} = \frac{m\lambda}{2}, \qquad m = 0, 1, 2.....$$

Special case: Newton's rings



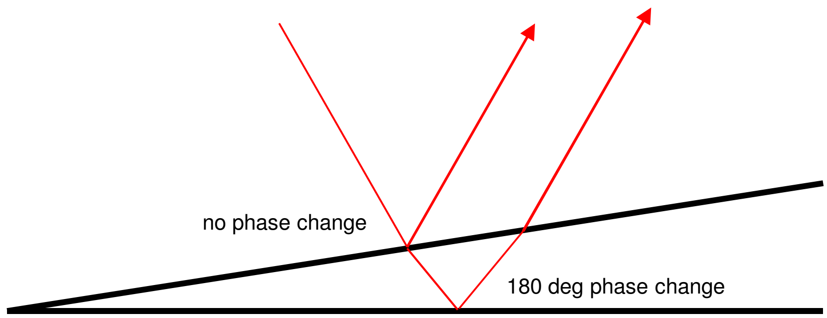


Same as microscope slide interference except that the change in thickness is is not linear and follows a very predictable shape.



Example

1) Two flat microscope slides, 10 cm long are touching on one side and are separated by 3 microns on the other. How many dark interference bands will appear on the slide if you look at the reflection for 450 nm light?



path condition for destructive interference

$$t = \frac{m\lambda}{2}, \qquad m = 0, \pm 1, \pm 2.....$$

$m = 0 \quad \Rightarrow \quad t = 0$

$m = 1 \quad \Rightarrow \quad t = 0.225$

$m = 2 \quad \Rightarrow \quad t = 0.450$

$m = 3 \quad \Rightarrow \quad t = ...$

a dark band occurs whenever the thickness changes by 0.225 microns.

$\frac{3}{0.225} = 13.33$, so the last band occurs when $m = 13$

$\therefore$ there are 14 dark bands (there is one for $m = 0$)