VS203B. Solutions for problem set #1

1. We need to satisfy the following two equations:

$$F_{doublet} = F_1 + F_2 \qquad CA_{doublet} = CA_1 + CA_2 = \frac{F_1}{\upsilon_1} + \frac{F_2}{\upsilon_2}$$

Solution: The refractive efficiencies for ophthalmic crown glass is 58.6 and for dense flint glass is 30.0. From the condition for zero aberration,

$$0 = \frac{F_1}{58.6} + \frac{F_2}{30.0} \qquad (1)$$

We also want to satisfy the condition: $F_1 + F_2 = -10$ (2)

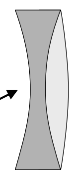
From equation (1): $F_1 = -\frac{58.6 \cdot F_2}{30.0}$

Sub equation (1) into (2):

$$F_2\left(-\frac{58.6}{30.0} + 1\right) = -10 \text{ D} \quad \Rightarrow \quad F_2 = 10.49 \text{ D}$$

Then it follows that: $F_1 = -20.49$ D

The lens might look something like this:



2. The focal length of an eye with power 60 D for the D-line is: 1.333/60 = 22.22 mm. The radius of curvature of the reduced eye refracting surface (cornea) =

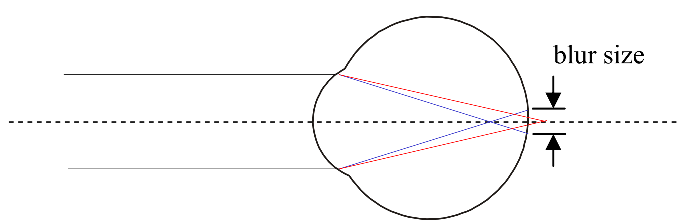
$$F_D = \frac{n_D - 1}{r} \therefore r = \frac{n_D - 1}{F_D} = \frac{0.333}{60} = 5.55 \text{ mm}$$

For red light:

$$f_c = \left(\frac{n_C}{n_C - 1}\right) r = 22.32 \text{ mm}$$

For blue light:

$$f_F = \left(\frac{n_F}{n_F - 1}\right) r = 22.01 \text{ mm}$$



Using similar triangles, we can derive the following equation to compute the blur size:

$$\frac{\text{blur size}}{\text{eye length-focal length}} = \frac{\text{pupil size}}{\text{focal length}} \Rightarrow \text{blur size} = \frac{(\text{eye length - focal length}) \times \text{pupil size}}{\text{focal length}}$$

For an 8 mm pupil the C-line and F-line blurs are: 36 and 76.3 microns respectively

For a 2 mm pupil the C-line and F-line blurs are: 9 and 19 microns respectively

You'll notice two things: First, the blue blur is larger then the red blur, which explains the appearance of a bluish halo around white lights. Second, when the pupil is large, the corresponding blurs are also large, which is why these phenomena are especially apparent at night.

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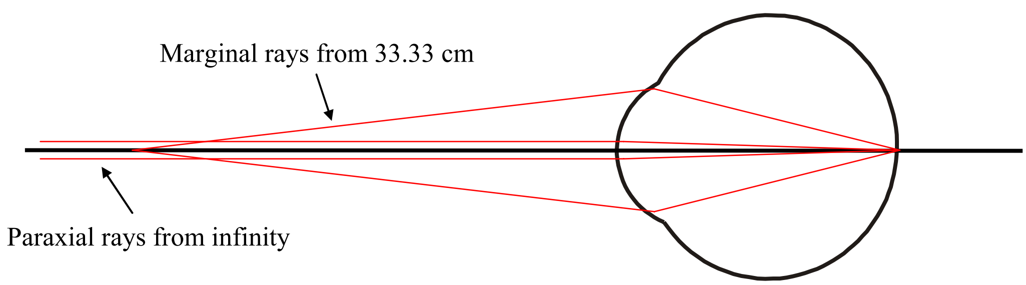
3. The power of the eye for paraxial rays is 0 D so the far point is at infinity. The power for the eye at the margin is +3D (due to the spherical aberration) so the far point is 1/3 = 33.33 cm in front of the eye. So the margins of the optical system of the eye can be used for near vision and the paraxial or more central part of the optical system can be used for distance.

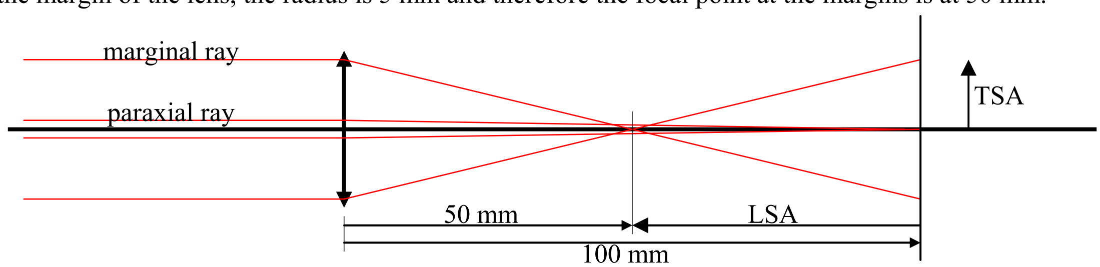


Do these lenses work? Marketers would argue that objects are equally clear at all distances. The skeptic would argue that objects are equally blurry at all distances. Depth of focus in increases but the cost is that there is no distance for which objects are very clear. Patient responses to these types of lenses are varied.

4. Substitute the values into the formula for spherical aberration to get:

$$f' = 100 - 2 \cdot r^2$$

At the margin of the lens, the radius is 5 mm and therefore the focal point at the margins is at 50 mm.



a) the power is higher in the margins so the type of aberration is positive spherical aberration

b) the longitudinal aberration is 50 mm positive spherical aberration

c) the longitudinal spherical aberration in D is:

$$\frac{1}{\text{marginal focus}} - \frac{1}{\text{paraxial focus}} = \frac{1}{0.05} - \frac{1}{0.1} = 10 \text{ D positive LSA}$$

d) the transverse spherical aberration is 5 mm. (just use similar triangles to solve this number)

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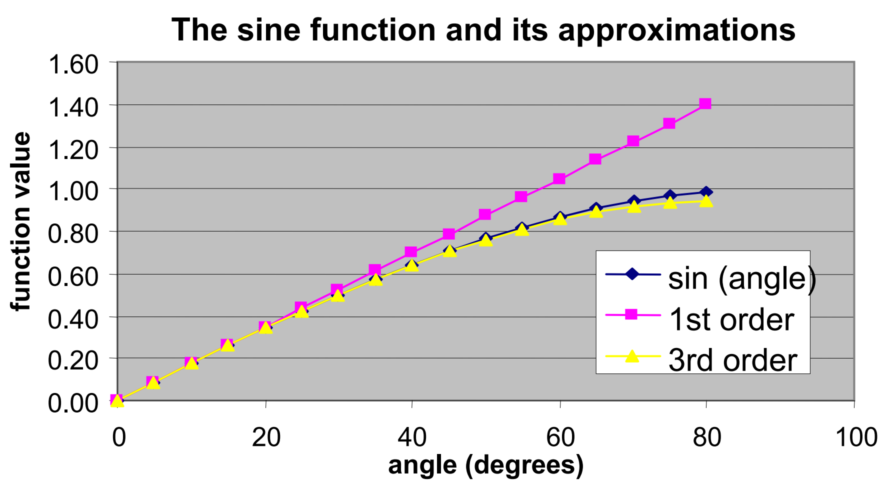
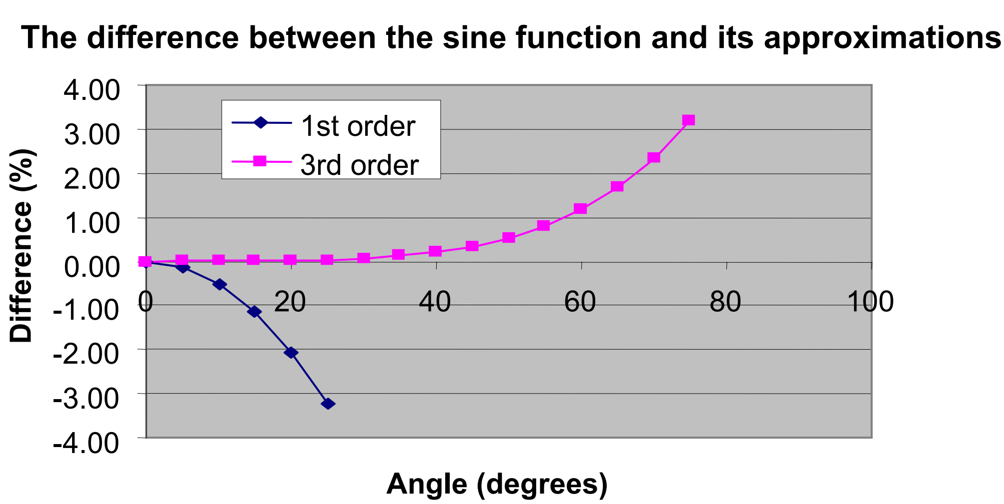
5.

		Exact	Approximations		Percentage differences	
angle in degrees	angle in radians	sin (angle)	1st order	3rd order	1st order	3rd order
0	0.00	0.00	0.00	0.00	0.00	0.00
5	0.09	0.09	0.09	0.09	-0.13	0.00
10	0.17	0.17	0.17	0.17	-0.51	0.00
15	0.26	0.26	0.26	0.26	-1.15	0.00
20	0.35	0.34	0.35	0.34	-2.06	0.01
25	0.44	0.42	0.44	0.42	-3.25	0.03
30	0.52	0.50	0.52	0.50	-4.72	0.07
35	0.61	0.57	0.61	0.57	-6.50	0.12
40	0.70	0.64	0.70	0.64	-8.61	0.21
45	0.79	0.71	0.79	0.70	-11.07	0.35
50	0.87	0.77	0.87	0.76	-13.92	0.54
55	0.96	0.82	0.96	0.81	-17.19	0.81
60	1.05	0.87	1.05	0.86	-20.92	1.18
65	1.13	0.91	1.13	0.89	-25.17	1.68
70	1.22	0.94	1.22	0.92	-30.01	2.33
75	1.31	0.97	1.31	0.94	-35.52	3.18
80	1.40	0.98	1.40	0.94	-41.78	4.29
19.71	0.34	0.34	0.34	0.34	-2.000	
67.65	1.18	0.92	1.18	0.91		2.000





6. For a 20 D lens, the angle at the margins is about 30 degrees. At an angle of about 30 degrees, the Gaussian approximation is off by almost 5 percent. This difference means that the lens makers formula will be close, but not exact. If you form an image with this lens, you will likely observe some blur due to aberrations.

7. Use the radial astigmatism equations here (*as long as the final angle is less than 20 degrees you will be OK*):

$$P_t = P\left[1 + \frac{4\phi^2}{3}\right]$$

$$P_s = P\left[1 + \frac{\phi^2}{3}\right]$$

The equivalent sphere must be -5.5 D, therefore…

$$\frac{P_t + P_s}{2} = -5.5D$$

$$= -2.5\left[1 + \frac{4\phi^2}{3} + 1 + \frac{\phi^2}{3}\right] = -2.5\left[2 + \frac{5\phi^2}{3}\right] = -5 - \frac{12.5\phi^2}{3}$$

$$\Rightarrow 0.5 = \frac{12.5\phi^2}{3}$$

$$\Rightarrow \phi = \sqrt{\frac{1.5}{12.5}} = 0.346 \text{ radians} = 19.84 \text{ degrees}$$

The lens must be tilted 19.84 degrees. Under these conditions, the astigmatism is the difference between the tangential and saggital meridians:

$$P_t = -5\left[1 + \frac{4(0.346)^2}{3}\right] = -5.80D$$

$$P_s = -5\left[1 + \frac{0.346^2}{3}\right] = -5.20D$$

$$P_t - P_s = -0.6D$$