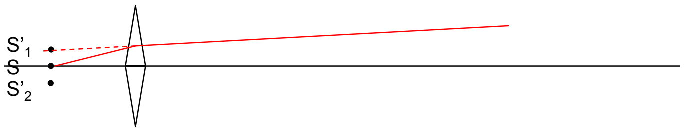
1.



The single source, S, appears to be be two sources when viewed through the pair of thin prisms. If the prism deviation is 1 cm at 1 m, then the deviation of one of the sources at 1 cm is:

$$1\text{ cm} \times 0.01 = 0.01\text{ cm} = 0.1\text{ mm}.$$

$$\overline{SS_1'} = 0.1\text{ mm and } \overline{SS_2'} = -0.1\text{ mm}$$

The apparent separation between the two point sources is:

$$\therefore \overline{S_2'S_1'} = 0.2\text{ mm}$$

separation between peaks is:

$$y = \frac{m\lambda s}{a}$$

$m = 1$ for first peak

$\lambda = 630\times10^{-9}$ m

$a = 0.2\times10^{-3}$ m

$s = 3.01$ m

$y = 9.5$ mm $\Leftarrow$ separation between two peaks

2.

$$y = \frac{m\lambda s}{a}$$

	peak locations (mm)		separation between peaks (mm)
	λ=630nm	λ=440nm	
a) m=0	0	0	0
b) m=1	9.48	6.62	2.86
c) m=2	18.96	13.24	5.72

VS203B Solutions for Diffraction, Interference and Resolution Problems

3.

$$y = \frac{m\lambda s}{a}$$

$\lambda = 630\times10^{-9}$

use $m = 1$ for distance to first peak

$s = 1$

$y = 1\times10^{-3}$ m

$a = \frac{m\lambda s}{y} = 0.63$ mm $\Leftarrow$ distance between the slits

4.

The aperture and diffraction pattern will be shaped as shown below because smaller slits produce broader diffraction patterns

Recall that with a slit aperture, the minima occur at:

$$y_{\min} = \frac{m\lambda s}{a} \quad \text{where } m = \pm1, \pm2....$$

The angular separation between the central peak to the first minimum of the diffraction pattern is:

$$\theta = \frac{\lambda}{a}$$

So the angular distance between the first two minima on either side of the central peak is

$$\theta_{\text{two minima}} = \frac{2\lambda}{a} \Rightarrow a = \frac{2\lambda}{\theta} \qquad \theta_{\text{two minima}} = \frac{2\lambda}{b} \Rightarrow b = \frac{2\lambda}{\theta}$$

for $\theta = \frac{\pi}{45}$ $(\theta = 4°)$, for $\theta = \frac{\pi}{90}$ $(\theta = 2°)$,

$a = 15.76\times10^{-6}$ m $= 15.76$ microns $\qquad b = 31.53\times10^{-6}$ m $= 31.53$ microns

5. With a circular aperture, the first minimum occurs at: $y_{\min} = \dfrac{1.22\lambda s}{a}$

If the diameter of the ring is 1 mm then the radius of the ring is 0.5 mm, therefore:

$$a = \frac{1.22 \times 450 \times 10^{-9} \times 2}{0.5 \times 10^{-6}} = 0.002196 \text{ m}$$

The size of the aperture is 2.2 mm in diameter.

6. This is a resolution problem, not a double slit problem!

$$y_{\min} = \frac{1.22 \cdot \lambda s}{a} = \frac{1.22 \cdot 586 \times 10^{-9} \cdot 0.5}{2.5 \times 10^{-3}} = 0.143 \text{ mm}$$

The separation between the slits is 0.143 mm

7. First calculate the thickness of the ARC

$$t_{dest} = \frac{1}{4} \times \frac{\lambda}{n_c} = \frac{1}{4} \times \frac{580 \times 10^{-9}}{1.38} = 105 \times 10^{-9} \text{m}$$

This is optimal for 580 nm light, but there will be a different phase change for 450 nm light. The phase change (in degrees) for 450 nm light is. Recall that in an ARC, the phase change between the two reflected waves is only due to path difference.

$$\text{phase difference in waves} = \frac{2 \times 105 \times 10^{-9}}{\left(450 \times 10^{-9} / 1.38\right)} = 0.644$$

$$\text{phase difference in degrees} = 0.644 \times 360 = 232^{\circ}$$

Now, calculate the amplitude of the reflectance from each surface.

$$\text{amplitude of reflected wave \#1, } A_1 = r_1 = \frac{n_c - n_{air}}{n_c + n_{air}} = \frac{1.38 - 1}{1.38 + 1} = 0.16$$

$$\text{amplitude of reflected wave \#2, } A_2 = r_1 = \frac{n_g - n_c}{n_g + n_c} = \frac{1.5 - 1.38}{1.5 + 1.38} = 0.042$$

Finally, calculate the coherent intensity of the sum of the waves, taking into account both reflected amplitudes and the phase difference between the waves

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta)$$
$$= 0.16^2 + 0.041^2 + 2 \times 0.16 \times 0.041 \times \cos(232)$$
$$= 0.0192 = 1.92\%$$

The reflected intensity for 450 nm light is 1.92 %

8. First calculate the thickness of the ARC

$$t_{dest} = \frac{1}{4} \times \frac{\lambda}{n_c} = \frac{1}{4} \times \frac{550 \times 10^{-9}}{1.38} = 99.6 \times 10^{-9} \text{m}$$

This coating is optimal for 550 nm light and so the phase difference is 180 degrees.

Now, calculate the amplitude of the reflectance from each surface.

$$\text{amplitude of reflected wave \#1, } A_1 = r_1 = \frac{n_c - n_{air}}{n_c + n_{air}} = \frac{1.38 - 1}{1.38 + 1} = 0.16$$

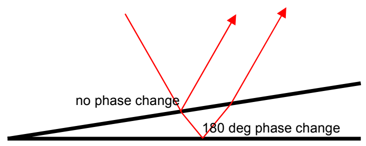
$$\text{amplitude of reflected wave \#2, } A_2 = r_1 = \frac{n_g - n_c}{n_g + n_c} = \frac{1.6 - 1.38}{1.6 + 1.38} = 0.074$$

Finally, calculate the coherent intensity of the sum of the waves, taking into account both reflected amplitudes and the phase difference between the waves

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta)$$
$$= 0.16^2 + 0.074^2 + 2 \times 0.16 \times 0.074 \times \cos(180)$$
$$= 0.0074 = 0.74\%$$

The reflected intensity for 550 nm light is 0.74 %

9.



The path condition for destructive interference is $t_{dest} = \dfrac{m}{2}\lambda, \qquad m = 0, \pm1, \pm2.....$

The dark bands occur when

$$m = 0 \Rightarrow t = 0$$
$$m = 1 \Rightarrow t = 0.225$$
$$m = 2 \Rightarrow t = 0.450$$
$$m = 3 \Rightarrow t = ...$$

Therefore a drak band is seen wheneverthe thickness is some multiple of 0.225 microns.

By the time the thickness gets to 3 microns, there will be

$$\frac{3}{0.225} = 13.33, \text{ so the last band occurs when } m = 13$$
$$\therefore \text{ there are 14 dark bands (there is one for } m = 0)$$

10.

a) $y_{\min} = \dfrac{1.22 \cdot \lambda s}{a} = \dfrac{1.22 \cdot 450 \times 10^{-9} \cdot 1}{5 \times 10^{-3}} = 0.11 \text{ mm}$

b) $y_{\min} = \dfrac{1.22 \cdot \lambda s}{a} = \dfrac{1.22 \cdot 650 \times 10^{-9} \cdot 1}{5 \times 10^{-3}} = 0.16 \text{ mm}$

c) $y_{\min} = \dfrac{1.22 \cdot \lambda s}{a} = \dfrac{1.22 \cdot 550 \times 10^{-9} \cdot 1}{1 \times 10^{-3}} = 0.67 \text{ mm}$

from a) $y_{\min} = \dfrac{1.22 \cdot 450 \times 10^{-9} \cdot 1}{5 \times 10^{-3}} = 0.11 \text{ mm} = \dfrac{1.22 \cdot 600 \times 10^{-9} \cdot 1}{a}$

$$\Rightarrow a = \frac{1.22 \cdot 600 \times 10^{-9} \cdot 1}{0.11 \times 10^{-3}} = 6.65 \text{ mm}$$

The pupil would have to be enlarged to 6.65 mm to get the same resolution in 600 nm light as a 5mm pupil in 450 nm light.